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Solitary wave solutions for the generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony nonlinear evolution equation

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Abstract

In this paper, we utilize the $\exp(-\varphi(\xi))$-expansion method to find exact and solitary wave solutions of the generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony nonlinear evolution equation. The generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony nonlinear evolution equation describes the model for the propagation of long waves that mingle with nonlinear and dissipative impact. This model is used in the analysis of the surface waves of long wavelength in hydro magnetic waves in cold plasma, liquids, acoustic waves in harmonic crystals and acoustic–gravity waves in compressible fluids. By using this method, seven different kinds of traveling wave solutions are successfully obtained for this model. The considered method and transformation techniques are efficient and consistent for solving nonlinear evolution equations and obtain exact solutions that are applied to the science and engineering fields.

Keywords: The $\exp(-\varphi(\xi))$-expansion method; The generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony nonlinear evolution equation; Traveling wave solutions; Solitary wave solutions.

1. Introduction

Two centuries ago, Zabusky and Kruskal discovered that many of our natural phenomena in different fields like mathematical physics, biology, chemistry, fluid mechanics, hydrodynamics, optics, and plasma physics and etc. can be represented as nonlinear partial differential equation or system. Zabusky and Kruskal are first researchers who give definite definitions of solitary waves that have several kinds like bright soliton (non-topological solution), dark soliton (topological solution), conidal wave, peakons, cuspons, stumpons and many other. Zabusky and Kruskal scrutinized the interaction of solitary waves and duplication of initial states.

Many researchers raced to discover some technics for solving such kind of nonlinear partial differential equations like the improved (G'/G)-expansion method, extended generalized Riccati equation, the modified simple equation method, generalized (G'/G)-expansion method, improved F-expansion method, the Exp-function method, the enhanced-expansion method, extended tanh-function method, $\exp(-i\phi(\xi))$-expansion method and so on [1–24].

As the objective of this article, we apply $\exp(-i\phi(\xi))$-expansion method on the generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony nonlinear evolution equation which considered as a regularized version of the KdV equation for shallow water waves. The basic mathematical difference between KdV and our model can be most facilely appreciated by comparing the squander relation in regard to linearized equations. It can be easily seen that these relations are comparable only for small wave numbers and they generate drastically different responses to short waves. The KdV equation describes

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long nonlinear waves of small amplitude on the surface of in viscid ideal fluid.

The rest of this paper is organized as follows: In Section 2, we give the description of the $\exp(-\varphi(\xi))$-expansion method. In Section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In Section 4, we give some figures that show the solitary wave solutions of equations and different types of it. In Section 5, conclusions are given.

2. Description of method

Let us assume we have the following nonlinear evolution equation

$$P(u, u_t, u_x, u_{tt}, u_{xx}, \ldots..) = 0, \tag{2.1}$$

where P is a polynomial in $u(x, t)$ and its partial derivatives. In the following, we give the main steps of this method.

Step 1. We use the traveling wave solution in the form

$$u(x,t) = u(\xi), \xi = x - ct, \tag{2.2}$$

where c is a positive constant, to reduce Eq. (2.1) to the following ODE:

$$Q\big(u, u', u'', u''', \ldots..\big) = 0, \tag{2.3}$$

where Q is a polynomial in $u(\xi)$ and its total derivatives.

Step 2. Suppose that the solution of ODE (2.3) can be expressed by a polynomial in $\exp(-\varphi(\xi))$ as follows

$$u(\xi) = a_m(\exp(-\varphi(\xi))^m + \ldots, a_m \neq 0, \tag{2.4}$$

where $\varphi(\xi)$ satisfies the ODE in the form

$$\varphi'(\xi) = \exp(-\varphi(\xi)) + \mu \exp(\varphi(\xi)) + \lambda. \tag{2.5}$$

Step 3. Substituting Eq. (2.4) along with Eq. (2.5) into Eq. (2.3) and collecting all the terms of the same power $\exp(-m\varphi(\xi))$, (m=0,1,2,3…) and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of it.

Step 4. Substituting these values and the solutions of Eq. (2.5) into Eq. (2.3) we obtain the exact solutions of Eq. (2.1).

It is to be noted here that the construction of the $\exp(-\varphi(\xi))-$expansion method is similar to the construction of the $(\frac{G'}{G})$-expansion. For better understanding of the duality of both methods we cite [25–27].

3. Application

Here, we will apply the $\exp(-i\varphi(\xi))$-expansion method described in Section 2 to find the exact traveling wave solutions and the solitary wave solutions of the generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony nonlinear evolution equation [28–30]. We consider the generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony nonlinear evolution equation.

$$u_t + u_x + \alpha(u^n)_x + \beta\big(u_{xt} + u_{yy}\big)_x = 0, \ \ n > 1, \tag{3.1}$$

where (α, β) are real constants. By using the wave transformation $u(\xi) = u(x, y, t)$, since $\xi = x + y - ct$, we get:

$$-cu' + u' + \alpha(u^n)' + \beta\big(-cu'' + u''\big)' = 0. \tag{3.2}$$

By integrating Eq. (3.2) and neglecting the constant of integration we obtain:

$$(1-c)u' + \alpha(u^n) + \beta(1-c)u'' = 0. \tag{3.3}$$

Balance the highest order derivatives and nonlinear terms appearing in Eq. (3.3) (u^n and $u'' \Rightarrow m = \frac{2}{n-1}$). So that we use transformation $[u = v^{\frac{2}{n-1}}]$ and substituting this transformation into Eq. (3.3) we get:

$$(1-c)(n-1)^2v^2 + \alpha(n-1)^2v^4 + \beta(1-c)(6-2n)v'^2 \\ +3\beta(1-c)(n-1)vv'' = 0. \tag{3.4}$$

Balance the highest order derivatives and nonlinear terms appearing in Eq. (3.4) (v^4 and $vv'' \Rightarrow m = 1$). So that, by using Eq. (2.4) we get the formal solution of Eq. (3.5)

$$v(\xi) = a_0 + a_1\exp(-\phi(\xi)). \tag{3.5}$$

Substituting Eq. (3.5) and its derivative into Eq. (3.4) and collecting all terms with the same power of $\exp(-4\phi(\xi))$, $\exp(-3\phi(\xi))$, $\exp(-2\phi(\xi))$, $\exp(-\phi(\xi))$, $\exp(0\phi(\xi))$ we get:

$$\begin{cases} \alpha(n-1)^2\big(a_1^4\big) + \beta(1-c)(6-2n)\big(a_1^2\big) \\ \quad +3\beta(1-c)(n-1)\big(2a_1^2\big) = 0, \\ \alpha(n-1)^2\big(4a_0a_1^3\big) + \beta(1-c)(6-2n)\big(2a_1^2\lambda\big) \\ \quad +3\beta(1-c)(n-1)\big(2a_0a_1 + 3a_1^2\lambda\big) = 0, \\ (1-c)(n-1)^2\big(a_1^2\big) + \alpha(n-1)^2\big(6a_0^2a_1^2\big) \\ \quad +\beta(1-c)(6-2n)\begin{pmatrix} 2a_1^2\mu \\ +a_1^2\lambda^2 \end{pmatrix} \\ \quad +3\beta(1-c)(n-1)\big(3a_0a_1\lambda + 2a_1^2\mu + a_1^2\lambda^2\big) = 0, \\ (1-c)(n-1)^2(2a_0a_1) + \alpha(n-1)^2\big(4a_0^3a_1\big) \\ \quad +\beta(1-c)(6-2n)\big(2a_1^2\mu\lambda\big) \\ \quad +3\beta(1-c)(n-1)\big(2a_0a_1\mu + a_0a_1\lambda^2 + a_1^2\mu\lambda\big) = 0, \\ (1-c)(n-1)^2\big(a_0^2\big) + \alpha(n-1)^2\big(a_0^4\big) \\ \quad +\beta(1-c)(6-2n)\big(a_1^2\mu\big) \\ \quad +3\beta(1-c)(n-1)(a_0a_1\lambda\mu) = 0. \end{cases} \tag{3.6}$$

Solving above system by using maple 16, we get:

$$n = 3, \alpha = \frac{-4(c-1)}{a_1^2\big(-\lambda^2 + 4\mu\big)}, \beta = \frac{-4}{3\big(-\lambda^2 + 4\mu\big)},$$

$$a_0 = \frac{a_1\lambda}{2}, \ \ a_1 = a_1.$$

Thus the solution is

$$v(\xi)=\frac{a_1\lambda}{2}+a_1\exp(-\phi(\xi)). \tag{3.7}$$

Now, we discuss the following cases:
When $\lambda^2-4\mu>0,\ \mu\neq 0$,

$$v(\xi)=\frac{a_1\lambda}{2}+\frac{2\ a_1\ \mu}{-\sqrt{\lambda^2-4\mu}\ tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\ (\xi+c_1)\right)-\lambda}, \tag{3.8}$$

$$u(x,y,t)=\left[\frac{a_1\lambda}{2}+\frac{2\ a_1\ \mu}{-\sqrt{\lambda^2-4\mu}\ tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\ ((x+y-ct)+c_1)\right)-\lambda}\right]^{\frac{2}{n-1}}, \tag{3.8a}$$

and

$$v(\xi)=\frac{a_1\lambda}{2}+\frac{2\ a_1\ \mu}{-\sqrt{\lambda^2-4\mu}\ coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\ (\xi+c_1)\right)-\lambda}, \tag{3.9}$$

$$u(x,y,t)=\left[\frac{a_1\lambda}{2}+\frac{2\ a_1\ \mu}{-\sqrt{\lambda^2-4\mu}\ coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\ ((x+y-ct)+c_1)\right)-\lambda}\right]^{\frac{2}{n-1}}. \tag{3.9a}$$

When $\lambda^2-\ 4\mu>0,\ \ \mu=0$,

$$v(\xi)=\frac{a_1\lambda}{2}+\frac{\lambda\ a_1}{\exp(\lambda(\xi+C_1))-1}, \tag{3.10}$$

$$u(x,y,t)=\left[\frac{a_1\lambda}{2}+\frac{\lambda\ a_1}{\exp(\lambda((x+y-ct)+C_1))-1}\right]^{\frac{2}{n-1}}. \tag{3.10a}$$

When $\lambda^2-4\mu=\ 0,\ \ \mu\neq\ 0,\lambda\neq 0$,

$$v(\xi)=\frac{a_1\lambda}{2}-\frac{a_1\lambda^2(\xi+C_1)}{2(\lambda(\xi+C_1)+2}, \tag{3.11}$$

$$u(x,y,t)=\left[\frac{a_1\lambda}{2}-\frac{a_1\lambda^2((x+y-ct)+C_1)}{2(\lambda((x+y-ct)+C_1)+2}.\right]^{\frac{2}{n-1}}. \tag{3.11a}$$

When $\lambda^2-4\mu=0,\ \ \mu\neq 0,\lambda=0$,

$$v(\xi)=\frac{a_1\lambda}{2}+\frac{a_1}{\xi+C_1}, \tag{3.12}$$

$$u(x,y,t)=\left[\frac{a_1\lambda}{2}+\frac{a_1}{(x+y-ct)+C_1}\right]^{\frac{2}{n-1}}. \tag{3.12a}$$

When $\lambda^2-4\mu<0$

$$v(\xi)=\frac{a_1\lambda}{2}+\frac{2\ a_1\ \mu}{\sqrt{4\mu-\lambda^2}tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ (\xi+c_1)\right)-\lambda}, \tag{3.13}$$

$$u(x,y,t)=\left[\frac{a_1\lambda}{2}+\frac{2\ a_1\ \mu}{\sqrt{4\mu-\lambda^2}tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ ((x+y-ct)+c_1)\right)-\lambda}\right]^{\frac{2}{n-1}}, \tag{3.13a}$$

and

$$v(\xi)=\frac{a_1\lambda}{2}+\frac{2\ a_1\ \mu}{\sqrt{4\mu-\lambda^2}cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ (\xi+c_1)\right)-\lambda}, \tag{3.14}$$

$$u(x,y,t)=\left[\frac{a_1\lambda}{2}+\frac{2\ a_1\ \mu}{\sqrt{4\mu-\lambda^2}cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ ((x+y-ct)+c_1)\right)-\lambda}\right]^{\frac{2}{n-1}}. \tag{3.14a}$$

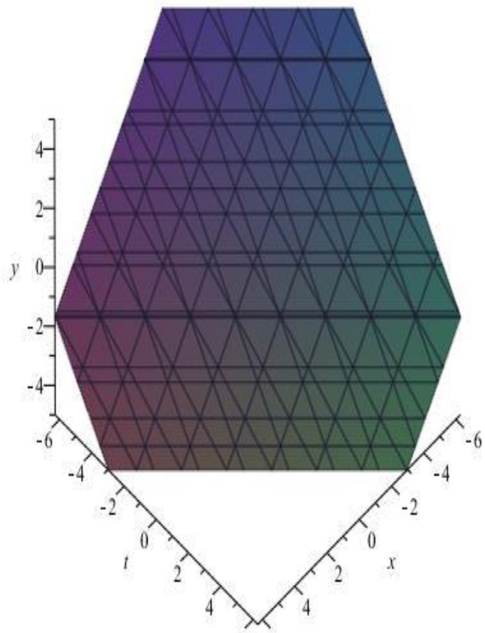
3.1. Note that

All the obtained results have been checked with Maple 16 by putting them back into the original equation and found correct.

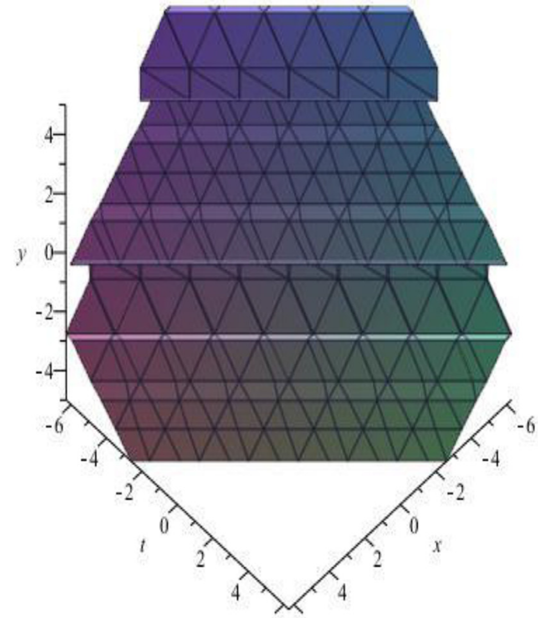
3.2. Discussion of the results

By applying here in this article the $\exp(-\varphi(\xi))$-expansion method to find exact and solitary wave solutions of the generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony nonlinear evolution equation we get seven different kinds of solitary traveling wave solutions. By comparison our results and other results that were obtained in [30], we get Eq. (3.9a) is equal to Eq. (42) when $[n=2,\ \ a_0=\lambda=0,\ \ a_1=\frac{4\ \beta(b^2-a^2)}{\alpha(8\ \ a^2\beta-1)}]$ and we get Eq. (3.9a) is equal to Eq. (48) when $[n=3,\ \ a_0=\lambda=0,\ \ a_1=\frac{-2\sqrt{2\beta(b^2-a^2)}}{\alpha(2\ \ a^2\beta-1)}]$ where all other results are considered to be a new form of solitary wave solution for the generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony nonlinear evolution equation.

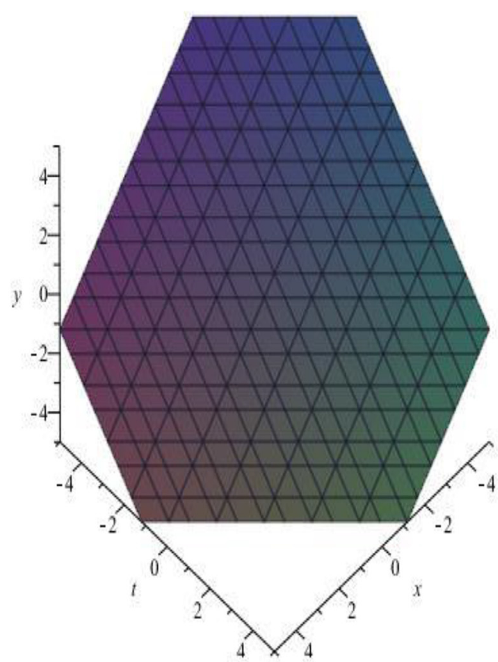
4. Figures



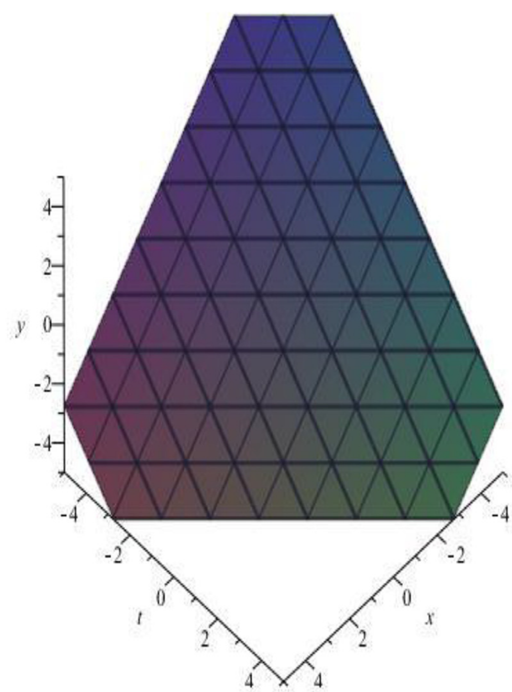
Solitary wave solutions of Eq. (3.8) when ($\lambda = 3, \mu = 2, C_1 = 1, c = -1, a_1 = 2$).



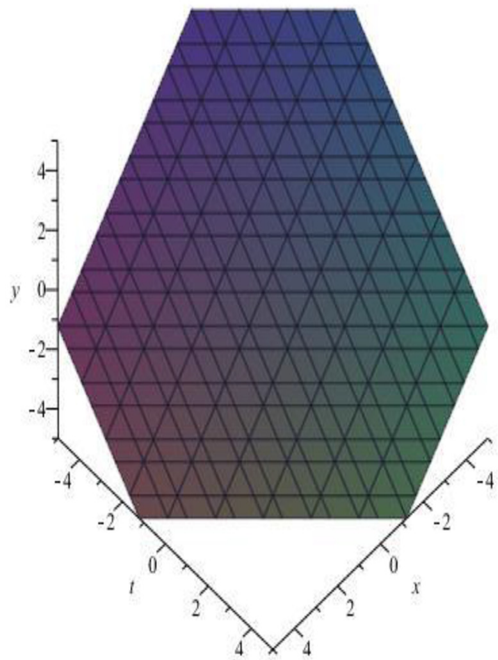
Solitary wave solutions of Eq. (3.9) when ($\lambda = 3, \mu = 2, C_1 = 1, c = -1, a_1 = 2$).



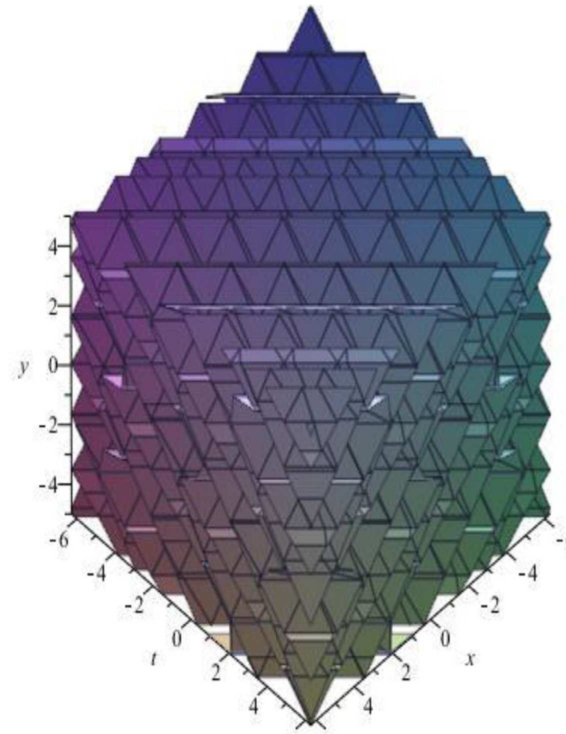
Solitary wave solutions of Eq. (3.10) when ($\lambda = 3, \mu = 0, C_1 = 1, c = -1, a_1 = 2$).



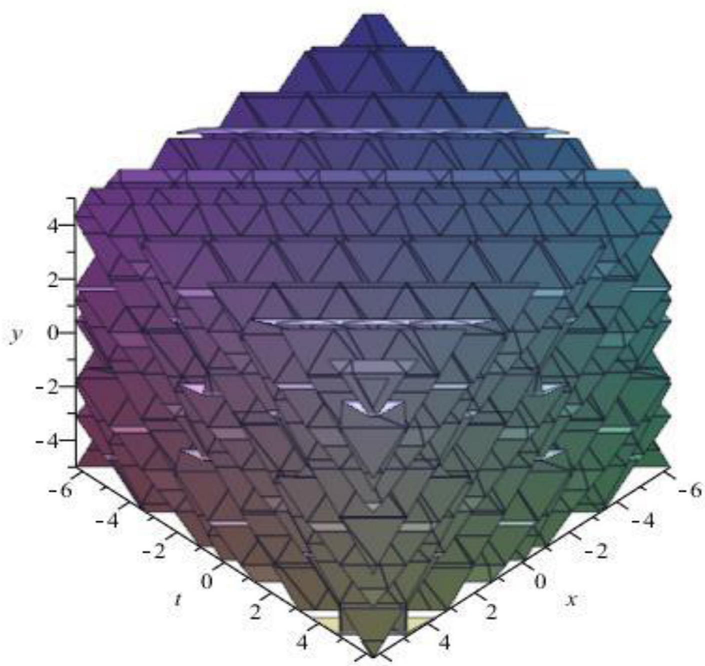
Solitary wave solutions of Eq. (3.11) when ($\lambda = 2\sqrt{2}, \mu = 2, C_1 = 1, c = -1, a_1 = 2$).



Solitary wave solutions of Eq. (3.12) when $(\boldsymbol{\lambda = \mu = 0, C_1 = 1, c = -1, a_1 = 2})$.



Solitary wave solutions of Eq. (3.13) when $(\boldsymbol{\lambda = 2\sqrt{2}, \mu = 3, C_1 = 1, c = -1, a_1 = 2})$.



Solitary wave solutions of Eq. (3.14) when $(\boldsymbol{\lambda = 2\sqrt{2}, \mu = 3, C_1 = 1, c = -1, a_1 = 2})$.

5. Conclusion

In this article, we succeeded in applying the $\exp(-\varphi(\xi))$-expansion method for the generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony nonlinear evolution equation. We obtained exact and solitary traveling wave solutions for our model and we also made a good comparison which showed that: Some of our results are similar to the results obtained by using different methods on the same model and not just like that, but also, we introduce new forms of solutions for the generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony nonlinear evolution equation. We plot our solutions to illustrate the solitary traveling wave solution shapes. Based on the above, we can say that: the

$\exp(-i\ \phi(\xi))$-expansion method is a very effective and powerful mathematical tool for applying on nonlinear partial differential equations with integer order and also with fraction order.

Competing interests

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. The author did not have any competing interests in this research.

Author's contributions

All parts contained in the research were carried out by the researcher through hard work and a review of the various references and contributions in the field of mathematics and the physical applied.

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