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Traveling wave solutions for shallow water equations

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Abstract

An extended homogeneous balance method is suggested in this paper. Based on computerized symbolic computation and the homogeneous balance method, new exact traveling wave solutions of nonlinear partial differential equations (PDEs) are presented. The shallow-water equations represent a simple yet realistic set of equations typically found in atmospheric or ocean modeling applications, we consider the exact solutions of the nonlinear generalized shallow water equation and the fourth order Boussinesq equation. Applying this method, with the aid of Mathematica, many new exact traveling wave solutions are successfully obtained.

Keywords: Extended homogeneous balance method; Shallow water equation; Boussinesq equation; Traveling wave solutions.

1. Introduction

The nonlinear equations are one of the most important phenomena across the world. Nonlinear phenomena have important effects on various fields of sciences, such as fluid mechanics, plasma physics, optical fibers, solid state physics, chemical kinematics, chemical physics and geochemistry. Explicit solutions to the mathematical modeling of physical problems are of fundamental importance. There are many methods in literature to solve the nonlinear equations, such as inverse scattering method [1,2], bilinear transformation [1,3], Hirota's method [4] the tanh-function method [5,6], extended tanh method [7,8], tanh-sech method [9], Differential transform method [10], sine-cosine method [11], Homotopy perturbation method [12], F-expansion method [13], general expansion method [14,15], and (G'/G) method [16–18] . The homogeneous balance (HB) method , which is a direct and effective algebraic method for the computation of exact traveling wave solutions, was first proposed by Wang [19,20]. Later [21,22], HB method is extended to search for other kinds of exact solutions not only the solitary one. Fan [23] used HB method to search for Bäcklund transformation and similarity reduction of nonlinear PDEs. Also, he showed that there is a close connection among the HB method, Weiss, Tabor, Carnevale (WTC) method and Clarkson, Kruskal (CK) method.

The shallow water equations (SWEs) are a system of hyperbolic PDEs describing fluid flow in the atmosphere, oceans, rivers and channels. SWEs describe fluid-flow-problems in a thin layer of fluid of constant density in hydrostatic balance, bounded from below by the bottom topography and from above by a free surface and derived from the physical conservation laws for the mass and momentum. The Boussinesq equations can be considered as an extension to the shallow water equations. Shallow water equations have been modeled to tsunamis predictions, atmospheric flows, storm surges, flows around structures (pier) and planetary flows.

The aim of this paper is to extend the homogeneous balance method to obtain more other kinds of exact solutions to nonlinear PDEs. The validity of the method is tested by its application to some nonlinear PDEs (The nonlinear generalized shallow water equation, and the fourth order Boussinesq equation). The obtained solutions include rational, periodical, singular, shock wave and solitary wave solutions.

In the following section, let us simply describe the extended homogeneous balance method.

2. Proposed analytical method

In general, consider a given PDE, say in two variables

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$$H(u,\ u_t,\ u_x,\ u_{xx},\ ....) = 0. \tag{1}$$

We seek for the special solution of Eq. (1), traveling wave solution, in the form

$$u(x,t) = u(\zeta), \quad \zeta = \ x - \lambda t, \tag{2}$$

where ϑ and L are constants to be determined later. Using the transformation (2), Eq. (1) reduces to a nonlinear ordinary differential equation (ODE). The next crucial step is that the solution we are looking for is expressed in the form

$$u(\zeta) = \sum_{i=0}^{n} a_i\ \omega^i + \sum_{i=1}^{n} b_i[1+\omega]^{-i}, \tag{3}$$

and

$$\omega' = k + M\ \omega + P\ \omega^2, \tag{4}$$

where a_i and b_i are constants, while k, M and P are parameters to be determined latter, $\omega = \omega(\zeta)$, and $\omega' = d\omega/d\zeta$. The mechanism for solitary wave solutions to occur is the fact that different effects (such as, the dispersion and nonlinearity) that act to change the wave forms in many nonlinear physical equations have to balance each other. Therefore, one may use the above fact to determine the parameter n which, must be a positive integer, can be found by balancing the highest-order linear term with the nonlinear terms. Substituting (3) and (4) in the relevant ODE will yield a system of ODEs with respect to a_0, a_i, b_i, k, M, P and λ (where $i = 1, \ldots, m$), because all the coefficients of ω^j (where $j = 0, 1, ...$) have to vanish. With the aid of *MATHEMATICA,* one can determine a_0, a_i, b_i, k, M, P and λ.

It is to be noted that the Riccati Eq. (4) can be solved using the homogeneous balance method as follows:

Case I: when P = 1, M = 0, the Riccati Eq. (4) has the following solutions

$$\omega = \begin{cases} -\sqrt{-k}\ \tanh[\sqrt{-k}\zeta], & \text{with } k < 0, \\ -\sqrt{-k}\ \coth[\sqrt{-k}\zeta], & \text{with } k < 0, \end{cases} \tag{5}$$

$$\omega = -\frac{1}{\zeta}, \quad \text{with } \ k = 0, \tag{6}$$

and

$$\omega = \begin{cases} \sqrt{k}\ \tan[\sqrt{k}\zeta], & \text{with } k > 0, \\ -\sqrt{k}\ \cot[\sqrt{k}\zeta], & \text{with } k > 0. \end{cases} \tag{7}$$

Since coth- and cot-type solutions appear in pairs with tanh- and tan-type solutions, respectively, they are omitted in this paper.

Case II:, Let $\omega = \sum_{i=0}^{m} A_i \tanh^i(p_1\zeta)$. Balancing ω' with ω^2 leads to

$$\omega = A_0 + A_1 \tanh(p_1\zeta) \tag{8}$$

Substituting Eq. (8) into (4), we have the following solution of Eq. (4)

$$\omega = -\frac{p_1}{2P}\tanh\left(\frac{p_1}{2}\zeta\right) - \frac{M}{2P}, \text{ with } Pk = \frac{M^2 - p_1^2}{4} \tag{9}$$

Similarly, let $\omega = \sum_{i=0}^{m} A_i \coth^i(p_1\zeta)$, then we obtain the following solution:

$$\omega = -\frac{p_1}{2P}\coth\left(\frac{p_1}{2}\zeta\right) - \frac{M}{2P}$$

with $Pk = \frac{M^2 - p_1^2}{4}$

Case III:, We suppose that the Riccati Eq. (4) have the following solutions of the form

$$\omega = A_0 + \sum_{i=0}^{m}(A_i f^i + B_i f^{i-1} g), \tag{10}$$

with

$$f = \frac{1}{\cosh\zeta + r}, \quad g = \frac{\sinh\zeta}{\cosh\zeta + r}, \tag{11}$$

Substituting Eqs. (10) and (11) into (4), we have the following solution of Eq. (4)

$$\omega = -\frac{1}{2P}\left(M + \frac{\sinh(\zeta) + \sqrt{r^2 - 1}}{\cosh(\zeta) + r}\right), \text{ with } Pk = \frac{M^2 - 1}{4} \tag{12}$$

where r is an arbitrary constant. It should be noticed that solution (12), as $r = 1$, degenerates to

$$\omega = -\frac{1}{2P}\left[M + \tanh\left(\frac{\zeta}{2}\right)\right] \tag{13}$$

Case IV:, We suppose that the Riccati Eq. (4) has the following solutions of the form

$$\omega = A_0 + \sum_{i=0}^{m} \sinh^{i-1}(A_i \sinh\eta + B_i \cosh\eta), \tag{14}$$

where $d\eta/d\zeta = \sinh\eta$ or $d\eta/d\zeta = \cosh\eta$ Balancing ω' with ω^2 leads to $m = 1$

$$\omega = A_0 + A_1 \sinh\eta + B_1 \cosh\eta. \tag{15}$$

when $d\eta/d\zeta = \sinh\eta$ we substitute (15) and $d\eta/d\zeta = \sinh\eta$ into (4) and set the coefficient of $\sinh^i\eta \cosh^j\eta$, $i = 0, 1, 2$, $j = 0, 1$ to zero and solve the obtained set of algebraic equations we get

$$A_0 = \frac{-M}{2P}, \quad A_1 = 0, B_1 = \frac{1}{P}, \tag{16}$$

where $k = \frac{M^2-4}{4P}$ and

$$A_0 = \frac{-M}{2P}, \quad A_1 = \pm\sqrt{\frac{1}{2P}}, \quad B_1 = \frac{1}{P}, \tag{17}$$

where $k = \frac{M^2-1}{4P}$. To $d\eta/d\zeta = \sinh\eta$ we have

$$\sinh\eta = -csch\zeta, \cosh\eta = -\coth\zeta. \tag{18}$$

From (16)-(18) we obtain

$$\omega = -\frac{M + 2\coth\zeta}{2P}. \tag{19}$$

where $k = \frac{M^2-4}{4P}$, and

$$\omega = -\frac{M \pm csch\zeta + \coth\zeta}{2P}. \tag{20}$$

where $k = \frac{M^2-1}{4P}$

3. Applications of the propsed method

In this section, we will illustrate the above approach for a class of nonlinear evolution equations namely, The nonlinear generalized shallow water equation and the fourth order Boussinesq equation.

3.1. Example 1. Shallow water equation

The shallow water equations are a set of hyperbolic partial differential equations that describe the flow below a pressure surface in a fluid

Let us first consider the nonlinear generalized shallow water equation

$$u_{xxxt}+\alpha u_x u_{xt}+\beta u_t u_{xx}-u_{xt}-\gamma u_{xx}=0, \tag{21}$$

where $\alpha,\ \beta,\ \gamma$ are constants. Applying the transformation $u(x,t)=U(\zeta), \zeta=x-\lambda t$ to Eq. (21) we find U satisfy the following ordinary differential equation

$$-\gamma U''+\lambda U''-\alpha\lambda U'U''-\beta\lambda U'U''-\lambda U^{(4)}=0 \tag{22}$$

Integrating (22) with respect to ζ once, we get

$$-\frac{1}{2}(\alpha+\beta)\lambda U'^2+(\lambda-\gamma)U'-\lambda U^{(3)}=0 \tag{23}$$

Balancing $U^{\prime\prime\prime}$ with U'^2 yields $|m|=1$. Therefore, we are looking for the solution in the form

$$U=a_0+b_0+a_1\omega+b_1(1+\omega)^{-1}. \tag{24}$$

Substituting Eqs. (24) and (4) in Eq. (23), we get a polynomial equation ω. Hence, equating the coefficient of ω^j (i.e., $j=0,1,2,...$) to zero and solving the obtained system of overdetermined algebraic equation using symbolic manipulation package MATHEMATICA, results in: The first set:

$$a_1=-\frac{12P}{\alpha+\beta},\quad P=\frac{M}{2},\quad k=\frac{1}{12}(12P+\alpha b_1+\beta b_1),\quad \alpha+\beta\neq 0,$$
$$b_1=-\frac{3}{4P(\alpha+\beta)},\quad \lambda\neq 0. \tag{25}$$

The second set:

$$a_1=0,\quad \alpha+\beta\neq 0,\quad k=\frac{M^2-1}{4P},$$
$$P\alpha\neq 0, b_1=\frac{12(k-M+P)}{\alpha+\beta},\quad P\neq 0. \tag{26}$$

Hence, for the first set we are left only with solutions satisfying cases II and III and IV. Since, the main criteria for these cases to be applicable is the compatibility condition,

$$Pk=\frac{M^2-p_1^2}{4}. \tag{27}$$

Therefore, substitute for P and k, from Eq. (25) into Eq. (27) and solve for p_1. It is found that

$$p_1=2\sqrt{\frac{\alpha}{16(\alpha+\beta)}+\frac{\beta}{16(\alpha+\beta)}}. \tag{28}$$

Therefore, solution to shallow water equation of the type (21), will be

$$u_1(x,t)=a_0+\frac{3\left(4p_1(M+2\tanh((x-\lambda t)p_1))+\frac{1}{p_1(M+2\tanh((x-\lambda t)p_1))-M}\right)}{2(\alpha+\beta)}, \tag{29}$$

and

$$u_2(x,t)=a_0+\frac{3\left(4(M+2\coth((x-\lambda t)p_1))p_1+\frac{1}{(M+2\coth((x-\lambda t)p_1))p_1-M}\right)}{2(\alpha+\beta)}, \tag{30}$$

In the same manner case III, results in the solution

$$u_3(x,t)=a_0+\frac{3(r+\cosh(x-\lambda t))}{2(\alpha+\beta)(\sinh(x-\lambda t)+\sqrt{r^2-1})}+\frac{6\left(M+\frac{\sinh(x-\lambda t)+\sqrt{r^2-1}}{r+\cosh(x-\lambda t)}\right)}{\alpha+\beta}, \tag{31}$$

with the condition that $p_1=1$,

For case IV, the solution form is

$$u_4(x,t)=a_0-\frac{6(M+\coth(x-\lambda t)+\operatorname{csch}(x-\lambda t))}{\alpha+\beta}+a_0-\frac{3}{2(\alpha+\beta)(2M+\coth(x-\lambda t)+\operatorname{csch}(x-\lambda t))}, \tag{32}$$

with the condition that $p_1=1$. and

$$u_5(x,t)=\frac{4(\alpha+\beta)a_0+3(8M+16\coth(x-\lambda t)+\tanh((x-\lambda t))}{4(\alpha+\beta)}, \tag{33}$$

with the condition that $p_1=2$.

For the second set (26), if $M=0,\ P=1$ we get the solutions satisfying case I for $k<0$. Therefore, the solution of shallow water equation of the type (21), will be

$$u_6(x,t)=a_0-\frac{12(k-M+P)}{(\alpha+\beta)(\sqrt{-k}\tanh(\sqrt{-k}(x-\lambda t))-1)}, \tag{34}$$

$$u_7(x,t)=a_0-\frac{12(k-M+P)}{(\alpha+\beta)(\sqrt{k}\coth(\sqrt{k}(x-\lambda t))-1)}, \tag{35}$$

Now for the solutions satisfying cases II and III and IV, we have the compatibility condition,

$$Pk=\frac{M^2-p_1^2}{4}. \tag{36}$$

Therefore, substitute for P and k, from Eq. (26) into Eq. (36) and solve for p_1. It is found that

$$p_1=2\sqrt{\frac{1}{4\beta}-\frac{k+1}{4\beta}}. \tag{37}$$

Hence, for case II, we get the following solutions:

$$u_8(x,t) = a_0 - \frac{6((M-2P)^2-1)}{(\alpha+\beta)(M-2P+2\tanh(x-\lambda t))}, \quad (38)$$

and

$$u_9(x,t) = a_0 - \frac{6\big((M-2P)^2-1\big)}{(\alpha+\beta)(M-2P+2\coth(x-\lambda t))}, \quad (39)$$

In the same manner case III, results in the solution

$$u_{10}(x,t) = a_0 + \frac{6}{\alpha+\beta}\left(2P + \frac{\sqrt{-1+r^2}}{r+\cosh[x-t\lambda]}\right), \quad (40)$$

with the condition that $p_1 = 1$.

For case IV, the solution form is

$$u_{11}(x,t) = a_0 + \frac{6(M^2-4PM+4P^2-1)}{(\alpha+\beta)(M+2P+\coth(x-\lambda t)+\operatorname{csch}(x-\lambda t))}, \quad (41)$$

with the condition that $p_1 = 1$.

3.2. Example 2. The Boussinesq equation

Consider The fourth order Boussinesq equation.

$$u_{tt} - A^2 u_{xx} + B(u^2)_{xx} + u_{xxxx} = 0, \quad (42)$$

where A and B are constants. Applying the transformation $u(x,t) = U(\zeta)$, $\zeta = x - \lambda t$ to Eq. (42) Then it is reduced to the following ordinary differential equation:

$$-A^2 U'' + \lambda^2 U'' - B(2U'^2 + 2UU'') + U^{(4)} = 0, \quad (43)$$

By integrating (33) with respect to ζ twice, we get

$$(\lambda^2 - A^2)U - BU^2 + U'' = 0 \quad (44)$$

Balancing U'' with U^2 yields m=2. Therefore, we are looking for the solution in the form

$$U = a_0 + b_0 + a_1\omega + b_1(1+\omega)^{-1} + a_2\omega^2 + b_2(1+\omega)^{-2}. \quad (45)$$

Substituting Eqs. (45) and (4) in Eq. (44), we get a polynomial equation ω. Hence, equating the coefficient of ω^j (i.e., $j = 0, 1, 2, \ldots$) to zero and solving the obtained system of overdetermined algebraic equation using symbolic manipulation package MATHEMATICA, results in two sets:

The first set:

$$M = 2P, \quad B \neq 0, \quad a_0 = \frac{6(2kP - P^2)}{B}, \quad a_1 = \frac{12P^2}{B},$$
$$b_1 = 0, a_2 = \frac{a_1}{2}, \quad b_2 = \frac{6(k^2 - 2Pk + P^2)}{B},$$
$$\lambda = \sqrt{A^2 - 4P^2 - 8kP + 2Ba_0},$$
$$k = \frac{3M^2 + 2Ba_0}{12M}, \quad kP - P^2 \neq 0. \quad (46)$$

The second set:

$$B \neq 0, \quad a_0 = \frac{M^2 + 2kP}{B}, \quad a_1 = \frac{6MP}{B}, \quad b_1 = 0,$$
$$a_2 = \frac{6P^2}{B}, \quad b_2 = 0, \quad \lambda = \sqrt{A^2 - M^2 - 8kP + 2Ba_0},$$
$$k = \frac{Ba_0 - M^2}{2P}, \quad P \neq 0. \quad (47)$$

For the first set, as in the previous example , we apply the compatibility condition in using the solutions satisfying cases II and III and IV.

$$Pk = \frac{M^2 - p_1^2}{4}. \quad (48)$$

Therefore, substitute for P and k, from Eq. (46), into Eq. (48) and solve for p_1. It is found that

$$p_1 = \frac{\sqrt{3M^2 - 2Ba_0}}{\sqrt{6}}. \quad (49)$$

Therefore, solution to the equation of the type (42), will be

$$u_1(x,t) = a_0 + \frac{3M^2(M-2k)^2}{2B(M - p_1(M + 2\tanh((x-\lambda t)p_1)))^2} + \frac{3p_1^2(M+2\tanh((x-\lambda t)p_1))^2}{2B} - \frac{3Mp_1(M+2\tanh((x-\lambda t)p_1))}{B}, \quad (50)$$

and

$$u_2(x,t) = a_0 + \frac{3M^2(M-2k)^2}{2B(M - (M + 2\coth((x-\lambda t)p_1))p_1)^2} + \frac{3(M+2\coth((x-\lambda t)p_1))^2 p_1^2}{2B} - \frac{3M(M+2\coth((x-\lambda t)p_1))p_1}{B}, \quad (51)$$

In the same manner case III, results in the solution

$$u_3(x,t) = a_0 + \frac{3M^2(M-2k)^2(r+\cosh(x-\lambda t))^2}{2B\left(\sinh(x-\lambda t) + \sqrt{r^2-1}\right)^2} + \frac{3\left(M + \frac{\sinh(x-\lambda t)+\sqrt{r^2-1}}{r+\cosh(x-\lambda t)}\right)^2}{2B} - \frac{3M\left(M + \frac{\sinh(x-\lambda t)+\sqrt{r^2-1}}{r+\cosh(x-\lambda t)}\right)}{B}, \quad (52)$$

with the condition that $p_1 = 1$.

For case IV, the solution form is

$$u_4(x,t) = a_0 + \frac{3M^2(M-2k)^2}{2B(2M + \coth(x-\lambda t) + \operatorname{csch}(x-\lambda t))^2} + \frac{3(M + \coth(x-\lambda t) + \operatorname{csch}(x-\lambda t))^2}{2B} + \frac{3M(M + \coth(x-\lambda t) + \operatorname{csch}(x-\lambda t))}{B}, \quad (53)$$

with the condition that $p_1 = 1$, and

$$u_5(x,t) = \frac{8Ba_0 + 3(((-2k)^2\tanh^2(x-\lambda t) - 4)M^2 + 16\coth^2(x-\lambda t))}{8B}, \quad (54)$$

with the condition that $p_1 = 2$.

For the second set (47), if $M = 0$, $P = 1$ we get the solutions satisfying case I. Therefore, for $k > 0$, the solution of Boussinesq equation of the type (42), will be

$$u_6(x,t) = a_0 + \frac{6k\tan^2(\sqrt{k}(x-\lambda t))}{B}, \tag{55}$$

and

$$u_7(x,t) = a_0 + \frac{6k\cot^2(\sqrt{k}(x-\lambda t))}{B}, \tag{56}$$

while for $k < 0$,

$$u_8(x,t) = a_0 - \frac{6k\tanh^2(\sqrt{-k}(x-\lambda t))}{B}, \tag{57}$$

$$u_9(x,t) = a_0 + \frac{6k\coth^2(\sqrt{k}(x-\lambda t))}{B}. \tag{58}$$

For $k = 0$,

$$u_{10}(x,t) = a_0 + \frac{6}{B\zeta^2}. \tag{59}$$

Now, for the solutions satisfying cases II and III and IV, we have the compatibility condition,

$$Pk = \frac{M^2 - p_1^2}{4}.$$

Therefore, substitute for P and k, from Eq. (47), and solve for p_1. It is found that

$$p_1 = \sqrt{3M^2 - 2Ba_0}. \tag{60}$$

Therefore, solution to the equation of the type (42), will be

$$u_{11}(x,t) = \frac{2Ba_0 + 3p_1(M + 2\tanh((x-\lambda t)p_1))(p_1(M + 2\tanh((x-\lambda t)p_1)) - 2M)}{2B}, \tag{61}$$

and

$$u_{12}(x,t) = \frac{2Ba_0 + 3(M + 2\coth((x-\lambda t)p_1))p_1((M + 2\coth((x-\lambda t)p_1))p_1 - 2M)}{2B}, \tag{62}$$

In the same manner case III, results in the solution

$$u_{13}(x,t) = a_0 + \frac{3(-\cosh(x-\lambda t)(2r+\cosh(x-\lambda t))M^2 - (M^2-1)r^2 + \sinh^2(x-\lambda t) + 2\sqrt{r^2-1}\sinh(x-\lambda t) - 1)}{2B(r+\cosh(x-\lambda t))^2}, \tag{63}$$

with the condition that $p_1 = 1$.

For case IV, the solution form is

$$u_{14}(x,t) = a_0 + \frac{3(M + \coth(x-\lambda t) + \operatorname{csch}(x-\lambda t))^2}{2B} + \frac{3M(M + \coth(x-\lambda t) + \operatorname{csch}(x-\lambda t))}{B}, \tag{64}$$

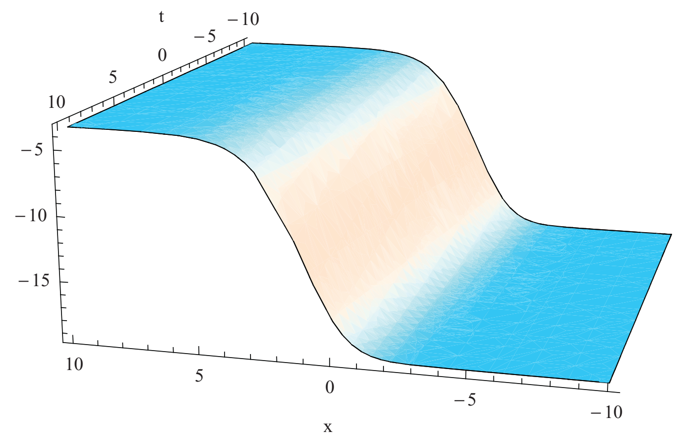

Fig. 1. Three-dimensional profile of the shock wave solution [given by Eq. (29)] for fixed values of $a_0 = 0.6$, $P = 1.5$, $\alpha = 0.1$, $\beta = 0.5$ and $\lambda = 0.1$.

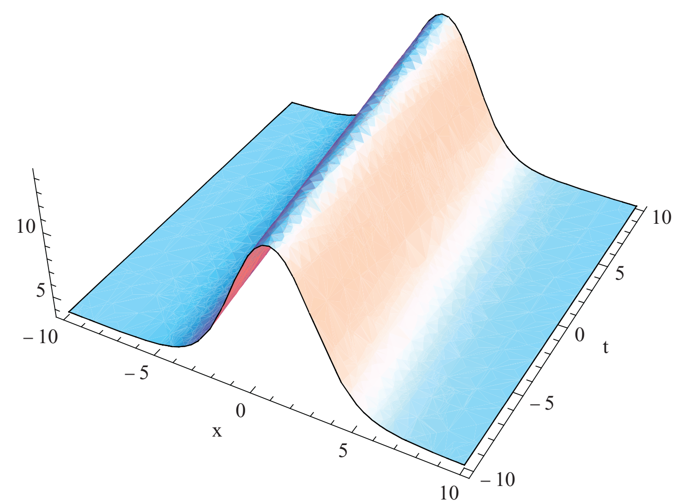

Fig. 2. Three-dimensional profile of the soliton wave solution [given by Eq. (40)] for $a_0 = 0.6$, $P = 1.5$, $\alpha = 0.1$, $\beta = 0.5$, $r = 3.2$ and $\lambda = 0.1$.

with the condition that $p_1 = 1$, and

$$u_{15}(x,t) = a_0 - \frac{3(M^2 - 4\coth^2(x-\lambda t))}{2B}, \tag{65}$$

with the condition that $p_1 = 2$.

In summary, the use of the extended homogeneous balance method gives rise to many traveling wave solutions that were formally derived for the nonlinear generalized shallow water equation and The fourth order Boussinesq equation. Some of them are new and interesting solutions, For example, as in solutions (29), (31) and (40) cannot be recovered using the tanh-method, the extended tanh method, and the (G'/G) method. These solutions include many types like rational, periodical, singular and solitary wave solutions which is very important to study the nonlinear properties of solitary waves. As example, the solution (29) is a shock wave solution as depicted in Fig. 1. Solutions (40), (57) are a bell-shaped solitary wave solution and represent the soliton solution, the profile of this solution is depicted in Fig. 2. Solution (55) is a sinusoidal-type periodical solution. Sinusoidal-type

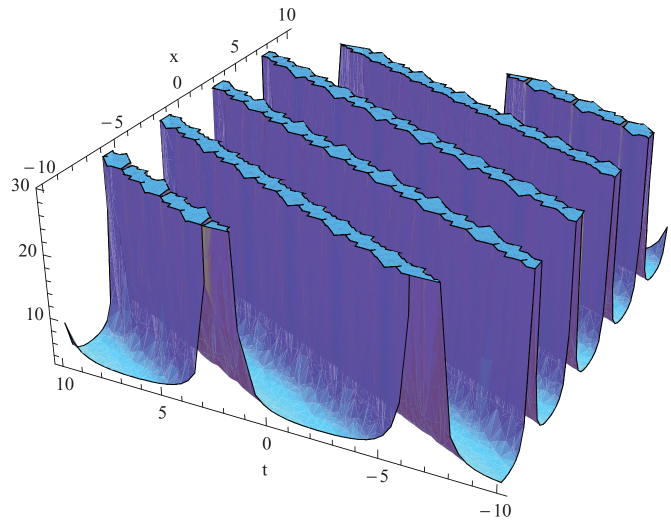

Fig. 3. Three-dimensional profile of the periodic solution [given by Eq. (55)] for $a_0 = 3.1$, $k = 0.5$, $\lambda = 0.1$ and $B = 3$.

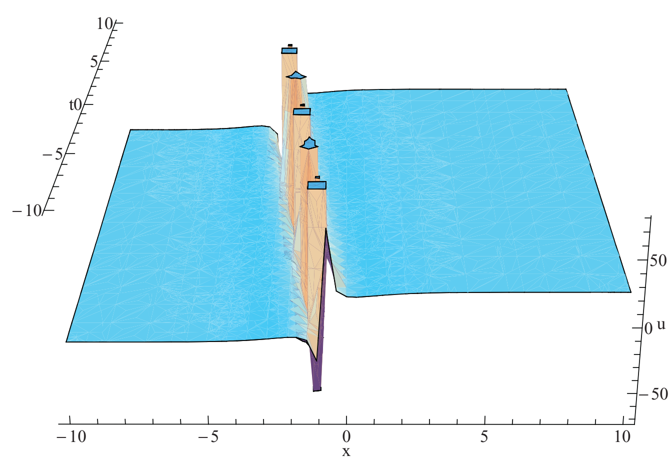

Fig. 4. Three-dimensional profile of the explosive/blowup pulse [given by Eq. (58)] for the same parameters as in Fig. 3.

periodical solutions develop a singularity at a finite point, i.e. for any fixed $t = t_0$ there exist an x_0 at which these solutions blow up , see Fig. 3, while solutions (31) and (58) are explosive/blowup solutions as depicted in Fig. 4. and the solutions in (59) is a rational-type solution. Rational solutions may be helpful to explain certain physical phenomena. The solutions are useful in physical aspects and applied mathematics.

4. Conclusions

An extended homogeneous balance method with computerized symbolic computation is developed to deal with nonlinear partial differential equations (PDEs). Traveling wave solutions were formally derived for the nonlinear generalized shallow water equation and The fourth order Boussinesq equation. This method can be also applied to other nonlinear evolution equations.

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